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13. ABSTRACT

There is a current moratorium on disposal of all excess Navy material with application to FG weapons systems. The material is being held in anticipation of future sales to FGs. Material holding costs preclude holding the material indefinitely. The purpose of this report is to present a mathematical model for computing the optimal amount of excess material to be held under economic criteria. The model was designed to provide management with a tool for computing the level of assets to hold in retention to maximize the financial return to the Navy through future sales to FGs. The model was also intended to be flexible in that input parameters may be varied and output solution values may be overridden to reflect management policy. It is anticipated that substantial financial savings will accrue through implementation of the model. The model will reduce material holding costs and free financial resources through liquidation (disposal) of assets with either no reasonable exception of demand or no economic justification for retention.

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AN ECONOMIC RETENTION MODEL FOR EXCESS NAVY ITEMS WITH FOREIGN MILITARY APPLICATIONS

OPERATIONS ANALYSIS DEPARTMENT

NAVY FLEET MATERIAL SUPPORT OFFICE

Mechanicsburg, Pennsylvania 17055

Report 132

AN ECONOMIC RETENTION MODEL FOR EXCESS NAVY ITEMS WITH FOREIGN MILITARY APPLICATIONS

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ABSTRACT

There is a current moratorium on disposal of all excess Navy material with application to FG (Foreign Government) weapons systems. The material is being held in anticipation of future sales to FGs. Material holding costs preclude holding the material indefinitely. The purpose of this report is to present a mathematical model for computing the optimal amount of excess material to be held under economic criteria.

The model was designed to provide management with a tool for computing the level of assets to hold in retention to maximize the financial return to the Navy through future sales to FGs. The model was also intended to be flexible in that input parameters may be varied and output solution values may be overridden to reflect management policy. It is anticipated that substantial financial savings will accrue through implementation of the model. The model will reduce material holding costs and free financial resources through liquidation (disposal) of assets with either no reasonable expectation of demand or no economic justification for retention.

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EXECUTIVE SUMMARY

1. Background. The United States has historically strived to preserve goodwill with friendly FGs, by providing maximum spare parts support subsequent to a weapons sale. In the early 1970s, a moratorium was imposed on disposal of long supply Navy assets with potential or actual FMS (Foreign Military Sales) usage. The moratorium was imposed for all assets with FMS usage because no criteria existed to selectively hold assets. The result of this policy was a large buildup of long supply assets at the ICPs (Inventory Control Points). These assets are held at an annual holding cost of approximately 21% to 23%. This investment in inventory also ties up funds which could be put to alternative uses if a portion of the assets could be liquidated (disposed) based on economic criteria. This report documents a mathematical model which can be used to compute the economically optimal amount of assets to hold in retention for potential FMS sale. Assets over this optimal level can then be disposed of without an adverse impact on sales revenues or a loss of logistics support to FGs over the economic retention period.

2. Objective. To design a mathematical model formulated on economic criteria which can be used to compute the optimal

amount of assets to hold in contingency retention for future sales to FGs under the FMS program.

3. Model Description. The FMS retention model computes the optimal amount of material to hold in retention by comparing revenues of sales to FGs with Navy holding costs. Assets will only be held as long as the revenue expected on a unit sold to a FG exceeds the cost of holding that unit. By using this criterion the optimal number of years for which assets should be held for sale to a FG can be determined. This optimal holding period may be translated into years of demand and ultimately a retention quantity.

Costs considered in the model formulation include physical storage costs and the opportunity cost, i.e., the cost of a foregone alternative, of not disposing of assets. Revenue considered is the financial resources to be gained by a sale of a unit to a FG. The model is flexible enough to be manipulated by varying input parameter settings or by overriding the output solution value to reflect management policy.

4. Approach. The model was developed to blend sound economic, mathematical, and inventory theory with the practical considerations of implementation. The model was evaluated by sensitivity analysis and the most sensitive variables identified. Model constraints were developed to factor model solution

values for real world considerations.

The model may be incorporated into the current UICP (Uniform Inventory Control Program) stratification programs as is the case with the current Navy economic retention model. The model may be solved by the well proven method of linear interpolation which can be incorporated into a subroutine of stratification (Application/Operation B20) programs.

5. Conclusions. The model provides NAVSUP (Naval Supply Systems Command) and the ICPs with a tool for computing the optimal amount of long supply assets to hold in retention for sale to FGs. Economic benefits expected to accrue through application of the model are an estimated $15M from reduced holding costs and $2M from increased returns from disposal.

I. INTRODUCTION

In recent years under United States foreign policy, Navy weapons systems and spare parts support have been sold to friendly FGs. Some of the weapons systems sold are obsolete and no longer in use by the U. S. Navy. Other weapons systems sold may be old U. S. Navy systems with little Navy usage or may be new weapons systems, such as the F-14.

A variety of support arrangements, i.e., FMS cases, have been used to supply spare parts from the Navy supply system to FGs using U. S. Navy weapons systems. These arrangements include COOPLOG (Cooperative Logistics) and DRP (Direct Requisitioning Procedure) cases (see APPENDIX A for definitions). To preserve goodwill between the U. S. and friendly FGs, to provide maximum spare parts support, and to maximize revenues from spare parts sales to FGs, a moratorium on disposal of Navy excess material with application to FG weapons systems was established in the early 1970s. Most of the material held under the moratorium applied to weapons systems already sold to FGs. However, some material was held to support mothballed aircraft not yet sold to FGs, but for which potential sales were expected, e.g., the S-2 aircraft. The result of the moratorium policy is a large amount of excess material held by the ICPs in contingency retention.

Reference 1 in APPENDIX B, tasked FMSO (Navy Fleet Material Support Office) to design a mathematical model for computing, under economic criteria, the amount of stock which should be held in contingency retention for items with FG application. The purpose of this report is to document the development of the economic retention model.

II. BACKGROUND

Excess items with FG application must be identified before an optimal retention level can be determined for these items. ASO and SPCC currently identify these items during the UICP stratification process. Excess items which apply to weapons systems not currently used by FGs must also be identified, since additional processing will be required to determine the optimal retention level. Some of the additional processing involves the development of a FG demand forecast. Knowledge of the price charged FGs for excess Navy material is also required to determine the optimal retention level. The following paragraphs provide more detail regarding these issues.

A. <u>IDENTIFICATION OF ITEMS WITH FG APPLICATION</u>. ASO uses the WSF (Weapons Systems File) in conjunction with the UICP stratification process to identify excess assets which are unique to aircraft designated for decreased Navy usage. ASO receives lists from NAVAIR (Naval Air Systems Command) and NAVSUP of aircraft designated for decreased Navy usage, but which have potential or actual FG usage. The WSF is used to identify items which have applications unique or peculiar to these aircraft, i.e., with no applications common to other U. S. Navy aircraft which will remain Navy active. The peculiar item lists are input to the UICP stratification pro-

cess which categorizes the excess assets of these items as a contingency retention requirement (see APPENDIX A for definition). The pertinent items held in contingency retention may be applicable to aircraft already in FG hands or mothballed aircraft expected to be sold to some FG.

SPCC also holds in contingency retention any excess assets for items unique to weapons systems designated for decreased Navy usage and FMS application as identified by the WSF and/or the THF (Transaction History File).

B. <u>DETERMINATION OF RETENTION LEVELS</u>. Assets held in contingency retention for weapons systems expected to be sold in the near future present a unique problem. The problem is how much excess assets should be held to provision a FG in the event of a future weapons system sale. No FG demand exists in this case to develop a demand forecast useful in driving a retention model. Also, no installed FG population data is available because the number of weapons systems to be sold is unknown. Thus, a provisioning problem exists with less than usual information. However, by making several basic assumptions, a FG demand forecast can be developed and the existing provisioning models can be used in lieu of developing new models:

1. Assume m (to be estimated) weapons systems will be procured by FGs.

2. Assume FG usage experience will equal Navy usage experience.

3. Assume FG aircraft will be flown for n (to be estimated) flying hours per month.

Under this assumption, a FG demand forecast necessary for input to the model described below will be developed and the Navy GRL/EL (Gross Requirements List/Equity List) model for ASO items and A/O (Application/Operation) E44 (International Logistics Support Products) model for SPCC items may be used to compute a provisioning quantity. The provisioning quantity can be expressed as years of demand (W), by dividing the provisioning quantity in units by the estimated FG demand forecast developed from Navy usage data. The amount of material that should be held in contingency retention is the maximum of the provisioning quantity or the quantity computed by the model described below.

Another situation exists where the items apply to weapons systems already used by FGs. These items may be unique to FG weapons systems or may be used on weapons systems common to the FG and the Navy and covered under COOPLOG or DRP support arrangements. Where Navy applications for common

items will be significantly reduced or phased out, excesses will develop due to decreasing Navy demand. The economic retention model developed in this report applies to excess assets of unique items on: (1) Navy weapons systems with slight usage or (2) terminal Navy weapons systems with expected or actual FG application for which FG demand data exists or a FG demand forecast could be developed from Navy usage rates under the above assumptions.

C. TREATMENT OF FG DEMAND. FG demand for an item is currently recorded in Navy UICP as either replenishable or nonreplenishable, depending on the FMS case covering the item. These recorded demands can be used to develop a FG demand forecast to be input to the model developed in this study.

1. Current Treatment of FG Demand. FG demand under COOPLOG cases is treated as replenishable (recurring) by Navy UICP. Demands recorded as replenishable demand observations are used to develop various demand forecasts. These forecasts are used to compute UICP levels, to develop budget projections in stratification, and as input to the Navy UICP economic retention model. As an exception, ASO does not treat COOPLOG demand as replenishable for repairable items, but establishes a fixed requirement based on nonreplenishable demand experience. Non-COOPLOG FG demands are treated as nonreplenishable by UICP and, therefore, are not used to develop Navy demand forecasts.

2. Development of a FG Demand Forecast. The development of a FG demand forecast is a prerequisite to utilizing the model developed in this study. The model should be run for applicable items subsequent to the Navy economic retention model in the UICP stratification program. The sum of the Navy and FG economic retention quantities would be held in contingency retention. Therefore, a FG demand forecast for two years after the budget year, which is compatible with D_g (Navy demand forecast for the second year after the budget year), is required for input to the FG economic retention model developed below. The two year history of all FG demand recorded on the UICP THF may be used to develop the initial FG forecast.

Many of the FG items are program-related. Demand forecasting of program-related items requires a knowledge of future flying hour programs. With few exceptions, FG flying hour programs are not readily available and the feasibility of obtaining them is questionable. As an alternative, a constant flying hour program may be assumed. Another alternative is to use exponential smoothing as the demand forecasting technique, i.e., use the forecasting technique for nonprogram-related items.

D. PRICING OF NAVY EXCESS ITEMS FOR SALE TO FOREIGN GOVERNMENTS. Reference 2 in APPENDIX B established DOD (Department of Defense) policy for determining item prices for sale of excess

material to FGs. The current Navy policy for sales of excess assets requisitioned by FGs is to charge the standard price plus administrative, accessorial, and assets use charges. Material which is excess and designated for disposal will be offered to FGs in a 'fire sale' at a percentage of the standard price as specified in reference 2. Material not saleable to FGs and designated for disposal is disposed of by the Defense Property Disposal Service. Non-RFI (Ready-for-Issue) material is sold at a discount price negotiated on an individual case basis.

III. TECHNICAL APPROACH

The technical approach taken in developing the model described below was to first make simplifying assumptions which would reflect the real world. The model is developed under these assumptions and is based on a fundamental economic tradeoff. Model constraints are developed to allow management to control the model output with lower and upper bounds. A model solution technique is identified which can produce an accurate solution value. A discussion is provided on procedures for determining model parameter values and on the results of a model sensitivity analysis.

A. <u>BASIC ASSUMPTIONS</u>. The following basic assumptions were made prior to developing the model:

1. A demand recording technique will be developed to record FMS demand separately from Navy demand and an FMS demand forecast comparable to D_g will be developed. Where future sales to FGs are anticipated, future demand may be estimated from Navy usage experience as described above in paragraph II.B.

2. Foreign government flying hour programs are constant. Also, current UICP exponential smoothing of demand for non-program-related items may be used to forecast FG demand.

3. The standard price charged at any year in the future will be the standard price in file today.

4. RFI assets will be preferred over non-RFI assets by FGs and will, therefore, be held in contingency retention before non-RFI assets.

5. An adequate demand history of two years exists in the UICP THF to develop a FG demand forecast for currently installed FG items.

6. Navy incurred costs for repair, shipping, packaging, storage, assets use, administrative, and all other charges of filling FG requisitions exactly equals Navy charges for repair, shipping, assets use, accessorial, administrative and other costs.

7. The annual obsolescence rate for an item is uniformly distributed over the expected life cycle of the item.

8. Every asset held for future sale to FGs as computed by the model will in fact be sold to a FG to fill a future requisition, i.e., the D_g forecast and model solution value are accurate.

9. Items held for sale to FGs will have zero salvage value.

The primary mathematical assumption made in this analysis was that the distribution of annual obsolescence rate for an item is uniformly distributed over the expected life cycle of the item. For example, if the expected life cycle of a given item were 10 years, the annual obsolescence rate would

be 1/10 years or .10 per year. The probability that an item is obsolete after t years may be found from the cumulative distribution or the relationship t years times the annual obsolescence rate. The probability of nonobsolescence after t years is therefore one minus the product of t years times the annual obsolescence rate (1-ta). FIGURE I of APPENDIX C is a graphical representation of the distribution of the obsolescence rate.

The assumption of a uniform distribution of annual obsolescence was used and is considered valid for two reasons:

- A precedent exists in that the Navy uses this assumption to derive the Navy obsolescence rate.
- No knowledge of each item's distribution of obsolescence exists and probably cannot be estimated by empirical analysis due to the uniqueness of each item.

B. MODEL DEVELOPMENT. The retention of excess material for FMS has, from a modeling perspective, similarities in economic costs to the retention of excess Navy material. For this reason, the Navy economic retention model described in reference 3, APPENDIX B, and the FMS model developed below have inherent conceptual similarities, and use many of the same factors in the retention decision. Whenever possible,

the same variable name was used to identify variables common to the existing Navy and newly developed FMS models.

1. Model Variables. The following variables are germane to the FMS economic retention model presented below.

DIMENSIONS	VARIABLE
Units	β = number of assets per item to be held in retention by the model
Fraction/Year	a = FG obsolescence rate for a given item
Dimensionless	b_1 = fraction of standard price at which RFI assets are sold to a FG on a requisition
Dimensionless	b_2 = fraction of standard price at which non-RFI assets are sold to a FG on a requisition
Dollars/Unit	c = standard price (DEN B053)
Dimensionless	g_1 = fraction of standard price charged FGs for RFI assets on a 'fire sale'
Dimensionless	g_2 = fraction of standard price charged FGs for non-RFI assets on a 'fire sale'
Fraction/Year	i = discount rate*
Dimensionless	p = disposal return rate*
Dimensionless	r_2 = repair survival rate (DEN F009)
Fraction/Year	s = storage cost*

*NOTE: The p, s, and i parameters may or may not equal

DIMENSIONS	VARIABLE
	their Navy model counterparts, depending on management policy. The FG obsolescence rate is distinct from the Navy obsolescence rate because most of the items have already become obsolete to the Navy but not to FGs.
Years	t_1 = maximum years of annual demand forecast for RFI assets to be held by the model
Years	t_2 = maximum years of annual demand forecast for non-RFI assets to be held by the model
Dimensionless	x = the fraction of assets of an item designated for disposal that are actually disposed, i.e., the probability of disposal
Dimensionless	y = the fraction of assets of an item designated for disposal but sold in a 'fire sale' vice disposed
Units/Year	D_g = annual FMS demand forecast for two years after the budget year (D_g is the sum of an estimated forecast for provisioned items and a computed forecast for items being currently demanded by FGs).
Units	ERL_c = Economic Retention Limit for consumables
Units	ERL_r = Economic Retention Limit for repairables

DIMENSIONS	VARIABLE
Years	R = shelf life
Years	W = years of demand of provisioning on a new sale* = $\frac{\text{provisioning quantity}}{D_g \text{ (from Navy usage)}}$
Years	Z_1 = minimum number of years of assets to be held for FMS (set by management)
Years	Z_2 = maximum number of years of assets to be held for FMS (set by management)

2. Basic Economic Tradeoff of the Model. Material should only be held for FMS under economic criteria when the proceeds of a future sale exceed the costs of holding the material in contingency retention. Holding costs include the opportunity cost (see APPENDIX A for definition) of not liquidating the assets through disposal and the physical storage costs. The material should be disposed of if:

Proceeds on disposal + storage costs >
proceeds of future sale to FG

FIGURE II of APPENDIX C is a graphical representation of this relationship. In this example, approximately 6.6 years of

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*NOTE: W is normally 1½ years of support for a COSMAL (Coordinated Shorebased Material Allowance List) or GRL (Gross Requirements List), although W may be any time period. W will equal 0 if no future weapons systems (on which the item is installed) sales are imminent.

FG demand for this item should be held in retention.

3. Mathematical Formulation of the Model. Mathematically, the proceeds on immediate disposal can be expressed by $p \cdot c$ (Opportunity cost of not liquidating the assets through disposal.) The annual storage costs can be expressed by $s \cdot c \cdot (1-ta)^*$. The variable t is the solution variable of the model. The factor (1-ta) is used because storage costs are only incurred on material which has not become obsolete and because the cumulative distribution of nonobsolescence is linear. The linear cumulative distribution follows from the assumed uniform (linear) distribution of obsolescence. Here $t = \frac{\beta}{D_g}$, which is the period of time the β^{th} unit will be held (number of years of stock based on demand). The discounted total holding costs may be expressed as:

$$p\ c + s\ c\ (1-a)\left(\frac{1}{1+i}\right) + s\ c\ (1-2a)\left(\frac{1}{1+i}\right)^2 + \ldots + s\ c\ (1-ta)\left(\frac{1}{1+i}\right)^t$$

$$\text{or } p\ c + s\ c \sum_{\ell=1}^{t} (1-\ell a) \left(\frac{1}{1+i}\right)^{\ell}$$

Some percentage y of the material designated for disposal may be offered and sold to FGs at a rate g.** The remainder

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*When the letter t is used without a subscript, it represents either t_1 or t_2.

**When the letter g is used without a subscript, it represents either g_1 or g_2.

of x will be disposed. Hence, there is a probability x of any unit being disposed and y of any unit being sold on a 'fire sale'. Therefore, the previous expression may be rewritten as:

$$x\ p\ c + y\ g\ c + s\ c \sum_{\ell=1}^{t} (1-\ell a)\left(\frac{1}{1+i}\right)^{\ell}$$

where

$$x + y = 1$$

The discounted expected proceeds on a sale to a FG on a requisition in year t under the above assumption of a uniform distribution of obsolescence may be expressed as:*

$$b\ c\ (1-ta)\left(\frac{1}{1+i}\right)^{t}$$

Mathematically expressed, material should be disposed when

$$x\ p\ c + y\ g\ c + s\ c \sum_{\ell=1}^{t} (1-\ell a)\left(\frac{1}{1+i}\right)^{\ell} > b\ c\ (1-ta)\left(\frac{1}{1+i}\right)^{t}$$

Therefore, after dividing the above expressions by c, the optimal level of stock that should be held is t_1 or t_2 years of annual demand when

$$x\ p + y\ g_1 + s \sum_{\ell=1}^{t_1} (1-\ell a)\left(\frac{1}{1+i}\right)^{\ell} = b_1\ (1-t_1 a)\left(\frac{1}{1+i}\right)^{t_1} \qquad (1)$$

for RFI assets or

*When the letter b is used without a subscript, it represents either b_1 or b_2.

$$x\,p + y\,g_2 + s \sum_{\ell=1}^{t_2} (1-\ell a)\left(\frac{1}{1+i}\right)^{\ell} = b_2\ (1-t_2 a)\left(\frac{1}{1+i}\right)^{t_2} \tag{2}$$

for non-RFI assets

Because the term $s \sum_{\ell=1}^{t_1} (1-\ell a)\left(\frac{1}{1+i}\right)^{\ell}$ allows only discrete values of t_1 or t_2, a continuous value for t_1 or t_2 cannot be found when t_1 or t_2 contains a fraction of a year. By assuming continuity, this problem can be overcome and

$$s \int_{\ell=0}^{t} (1-\ell a)\left(\frac{1}{1+i}\right)^{\ell} d\,\ell \approx s \sum_{\ell=1}^{t} (1-\ell a)\left(\frac{1}{1+i}\right)^{\ell}$$

The above integral can be integrated by parts, thus yielding

$$s \int_{\ell=0}^{t} (1-\ell a)\left(\frac{1}{1+i}\right)^{\ell} d\,\ell =$$

$$\frac{\left(\frac{1}{1+i}\right)^{t}\left[(1-ta)\ln\left(\frac{1}{1+i}\right) + a\right] - \left[\ln\left(\frac{1}{1+i}\right) + a\right]}{\left[\ln\left(\frac{1}{1+i}\right)\right]^2}$$

Equations (1) and (2) above then become by substitution

$$x\,p + y\,g_1 + s\left[\frac{\left(\frac{1}{1+i}\right)^{t_1}\left[(1-t_1 a)\ln\left(\frac{1}{1+i}\right) + a\right] - \left[\ln\left(\frac{1}{1+i}\right) + a\right]}{\left[\ln\left(\frac{1}{1+i}\right)\right]^2}\right] \tag{3}$$

$$= b_1\ (1-t_1 a)\left(\frac{1}{1+i}\right)^{t_1}$$

$$x\,p + y\,g_2 + s\left[\frac{\left(\frac{1}{1+i}\right)^{t_2}\left[(1-t_2a)\ln\left(\frac{1}{1+i}\right) + a\right] - \left[\ln\left(\frac{1}{1+i}\right) + a\right]}{\left[\ln\left(\frac{1}{1+i}\right)\right]^2}\right] \tag{4}$$

$$= b_2\ (1-t_2a)\left(\frac{1}{1+i}\right)^{t_2}$$

Mathematically expressed, the retention limit should be

$D_g t_1$ for RFI repairable assets and consumables

$D_g t_2$ for non-RFI assets

C. MODEL CONSTRAINTS. Real world factors such as limited shelf life and repair survival rates make constraints on the computed retention limit essential. Different constraints are required for consumable and repairable items as follows:

1. Consumables. Management may wish to institute a policy of holding more or less assets than computed by the model or constraining the retention level between upper and lower bounds. This may be accomplished by manipulating the Z_1 and Z_2 parameters. Also, some items are subject to limited shelf life of R years. Therefore, the final ERL relationship for consumable items is:

$$ERL_C = \text{Min}\ [RD_g;\ \text{Max}(Z_1D_g;\ WD_g;\ t_1D_g);\ Z_2D_g]^+$$

where

RD_g = maximum units which can be held due to shelf life limitations

t_1D_g = RFI units computed by the model as the optimal

retention level

WD_g = units expected to be sold to a FG on a provisioning agreement

Z_1D_g = minimum units acceptable for holding by management

Z_2D_g = maximum units acceptable for holding by management

2. <u>Repairables</u>. Again, management may wish to hold more or less assets than computed by the model.

a. If all assets of an item are RFI, then

$$ERL_r = \text{Min}\ [\text{Max}(Z_1D_g;\ WD_g;\ t_1D_g);\ Z_2D_g]^+ .$$

NOTE: ERL_r in this case equals ERL_c.

b. If all assets of an item are non-RFI, then

$$ERL_r = \text{Min}\ \left[\text{Max}\left(Z_1\ \frac{D_g}{r_2};\ W\frac{D_g}{r_2};\ t_2D_g\right);\ Z_2\ \frac{D_g}{r_2}\right]^+$$

where

$Z_1\ \frac{D_g}{r_2}$ = minimum units acceptable for holding by management

$W\ \frac{D_g}{r_2}$ = units expected to be sold to a FG on a provisioning agreement

t_2D_g = units of non-RFI assets computed by the model as the optimal retention level

$Z_2 \dfrac{D_g}{r_2}$ = maximum units acceptable for holding by management

The $Z_1 \dfrac{D_g}{r_2}$ and $Z_2 \dfrac{D_g}{r_2}$ expressions were derived as follows. Suppose management wishes to hold non-RFI assets to fill FG requisitions for K units. Then e non-RFI assets would be required to fill these requisitions such that

$$e \cdot r_2 = K \text{ or } e = \frac{K}{r_2} \ .$$

Expressing K units as $Z_1 D_g$ or $Z_2 D_g$, then

$$e = \frac{Z_1 D_g}{r_2} \text{ or } e = \frac{Z_2 D_g}{r_2}$$

c. If some assets of an item are RFI and some are non-RFI:

$$ERL_r = \text{Min} \left[\begin{array}{l} \text{Max} \left\{ \begin{array}{l} \text{minimum RFI units acceptable for holding by} \\ \text{management plus the minimum non-RFI units} \\ \text{acceptable for holding by management} \\ \\ \text{RFI units expected to be sold to a FG} \\ \text{on a provisioning agreement plus the non-} \\ \text{RFI units expected to be sold to a FG on} \\ \text{a provisioning agreement} \\ \\ \text{RFI units computed by the model as the} \\ \text{optimal retention level plus the factored} \\ \text{non-RFI units computed by the model as} \\ \text{the optimal retention level} \end{array} \right. \\ \\ \qquad \begin{array}{l} \text{the maximum RFI units acceptable for} \\ \text{holding by management plus the maximum} \\ \text{non-RFI units acceptable for holding by} \\ \text{management} \end{array} \end{array} \right]$$

Expressed mathematically:

$$ERL_r = \text{Min}\left[\begin{array}{l} \text{Max}\left\{\begin{array}{l} \left[\min(Z_1D_g;\ \text{RFI assets available})\ \text{RFI assets} + \max\left(0;\ \dfrac{Z_1D_g - \text{RFI assets available}}{r_2}\right)\text{non-RFI assets}\right] \\ \left[\min(WD_g;\ \text{RFI assets available})\ \text{RFI assets} + \max\left(0;\ \dfrac{WD_g - \text{RFI assets available}}{r_2}\right)\text{non-RFI assets}\right] \\ \left[\min(t_1D_g;\ \text{RFI assets available})\ \text{RFI assets} + \max\left(0;\left(\dfrac{t_2}{t_1}\right)(t_1D_g - \text{RFI assets})\right)\text{non-RFI assets}\right] \end{array}\right. \\ \left[\min(Z_2D_g;\ \text{RFI assets available})\ \text{RFI assets} + \max\left(0;\ \dfrac{Z_2D_g - \text{RFI assets available}}{r_2}\right)\text{non-RFI assets}\right] \end{array}\right]$$

where

$\min(Z_1D_g;$ RFI assets available) RFI assets = minimum RFI units acceptable for holding by management

$\max\left(0;\ \dfrac{Z_1D_g - \text{RFI assets available}}{r_2}\right)$ non-RFI assets = minimum non-RFI units acceptable for holding by management

$\min(WD_g;$ RFI assets available) RFI assets = RFI units expected to be sold to a FG on a provisioning agreement

$\max\left(0; \dfrac{WD_g\text{-RFI assets available}}{r_2}\right)$ non-RFI assets = non-RFI units expected to be sold to a FG on a provisioning agreement

$\min(t_1 D_g;$ RFI assets available$)$ RFI assets = RFI units computed by the model as the optimal retention level

$\max\left(0; \left(\dfrac{t_2}{t_1}\right)(t_1 D_g\text{-RFI assets})\right)$ non-RFI assets = the factored non-RFI units computed by the model as the optimal retention level

$\min(Z_2 D_g;$ RFI assets available$)$ RFI assets = maximum RFI units acceptable for holding by management

$\max\left(0; \dfrac{Z_2 D_g\text{-RFI assets available}}{r_2}\right)$ non-RFI assets = maximum non-RFI units acceptable for holding by management

For example: Suppose:

D_g = 10 units per year

RFI assets available = 100 units

Non-RFI assets available = 50 units

Z_1 = 12 years $\rightarrow$ $Z_1 D_g$ = 120 units

r_2 = .50

W = two years $\rightarrow$ WD_g = 20 units

t_1 = four years $\rightarrow$ $t_1 D_g$ = 40 units

t_2 = two years

Z_2 = 30 years $\rightarrow Z_2 D_g$ = 300 units

Then:

$$ERL_r = \text{Min}\left[\text{Max}\left\{\begin{array}{l}\left[\text{min}(120;100)\text{ RFI assets + max}\right.\\ \left.\left(0;\ \frac{120-100}{.5}\right)\text{ non-RFI assets}\right]\\ \left[\text{min}(20;100)\text{ RFI assets +}\right.\\ \left.\text{max}\left(0;\ \frac{20-100}{.5}\right)\text{ non-RFI assets}\right]\\ \left[\text{min}(40;100)\text{ RFI assets +}\right.\\ \left.\text{max}\left(0;\ \left(\frac{2}{4}\right)(40-100)\right)\text{ non-RFI assets}\right]\end{array}\right.\right.$$

$$\left.\left[\text{min}(300;100)\text{ RFI assets + max}\left(0;\ \frac{300-100}{.5}\right)\text{ non-RFI assets}\right]\right]^{+}$$

ERL_r = 100 RFI assets + 40 non-RFI assets = 140 assets.

3. The Z_1 and Z_2 parameters may be set to put lower and upper bounds on the values of ERL_c and ERL_r by setting $Z_2 > Z_1$. If Z_1 is set equal to Z_2, then Z_1 and Z_2 will act as effective model overrides because the ERL_c and ERL_r quantities will always equal Z_1.

D. MODEL SOLUTION TECHNIQUES. APPENDIX D describes a model solution technique and provides an example of its usage. The technique for finding the solution values (t_1,t_2) is known as the method of linear interpolation (as described

in reference 4 of APPENDIX B). Eleven iterations were required to find a t_1 solution value for the example used. The solution technique may be incorporated into a FORTRAN IV subroutine of the appropriate UICP stratification program. Other iterative solution techniques are available which are similar to the method of linear interpolation. However, the method of linear interpolation has an advantage in simplicity of application and ease of comprehension and is, therefore, recommended as the appropriate solution technique.

E. DETERMINATION OF PARAMETER VALUES. The model developed above contains a variety of parameter inputs. Some of these values will be set to specific values based on policy, and others will be set based on empirical evidence. The following comments will treat each parameter individually.

The x and y parameter values could be determined by empirical analysis by taking a sample of items designated for disposal. The ratio of assets of these items sold to FGs on a fire sale can then be determined and x and y computed as follows:

$$y = \frac{\text{number of assets of sample items sold on 'fire sale'}}{\text{number of assets of sample items to be sold on 'fire sale' or disposed}}$$

and $x = 1 - y$.

The p (disposal return rate) value can be determined by sampling the prices received for assets disposed and computing:

$$p = \frac{\text{average price received for items liquidated by disposal}}{\text{average standard price of items liquidated by disposal}}$$

The g_1 and g_2 values can be computed in accordance with reference 2 of APPENDIX B. For example, ASO previously charged a 'fire sale' price of 50% of standard price for RFI assets in A condition and 20% of standard price for non-RFI assets in F and G condition. Under this policy, $g_1 = .50$ and $g_2 = .20$.

Obsolescence rate for FGs can be computed for each item by estimating the useful life of each item and by assuming that the probability distribution of annual obsolescence is uniformly distributed. The following computation then applies:

$$a = \frac{1}{\text{useful life of the item in years}}$$

The fraction of standard price at which RFI or non-RFI assets are sold to FGs may be fixed rates set for all items by policy. In this case, b_1 and b_2 would equal these respective policy rates. If the rates vary by item, empirical data and the following relationships should be used:

$$b_1 = \frac{\text{average price received for RFI assets of item sold to FGs}}{\text{standard price of item sold to FGs}}$$

$$b_2 = \frac{\text{average price received for non-RFI assets of item sold to FGs}}{\text{standard price of item sold to FGs}}$$

From a theoretical viewpoint, the discount rate and storage cost should be the same values for the FMS model as are used for the Navy model. However, management may wish to vary these parameters to reflect policy.

The shelf life R, the standard price c, and the repair survival rate r_2 may be obtained from UICP stratification files. The parameters Z_1, Z_2, and W will be set by management policy.

F. <u>MODEL EVALUATION BY SENSITIVITY ANALYSIS</u>. The models shown above as equations (3) and (4) of paragraph III.B, were evaluated by sensitivity analysis. In this approach, benchmark values for the model parameters were chosen and t was computed. Then, one or more of the parameters were varied, and the resultant t values were compared to the t value computed as a benchmark. The results of this analysis are shown in APPENDIX E. The parameter values were chosen for the sensitivity analysis to reflect real world conditions. For example, it is inconceivable that an investor could earn more than 30% interest in today's economy. Therefore, "i" (the interest rate) was varied only up to 30%. Also, g may equal only 5%, 10%, 20%, 35%, or 50% as specified by reference 2 of APPENDIX B.

The sensitivity of the net proceeds of FG sales to t years of demand held in retention may be seen in FIGURE III of APPENDIX C. The increasing slope of the curves of FIGURE III of APPENDIX C represents increasing sensitivity, because changes in the independent variable result in greater than corresponding changes in the dependent variable. The net proceeds to be gained on a sale to a FG become increasingly sensitive as t decreases, but are relatively insensitive to small reductions in t below the optimal 6.69 value. Therefore, management may hold a little less than the economically optimal number of years of demand with a relatively small impact on net proceeds. As t becomes smaller, progressively greater than corresponding decreases in net proceeds occur.

The sensitivity of t to variations in one or more model variables may be seen in FIGURES IV through X of APPENDIX C. The steepness of the slopes of the curves represents the sensitivity of t to each variable changed. Steeper slopes represent increasing sensitivity. The t value showed the highest sensitivity in almost all cases when the values of the independent variables were relatively small. The one exception occurred with the variable x, which produced the greatest percentage change in t when x was relatively

large, i.e., near 1.00. It is anticipated that most of the model parameters used will be small, e.g., i = .10, a = .10, s = .01. Therefore, care should be taken when estimating these parameters in actual production usage of the model because small parameter errors will produce greater percentage errors in the solution value of t.

The model solution value also becomes more sensitive as the price (b c) charged FGs for assets used to fill requisitions approaches the price received by the Navy on a 'fire sale' or disposal action (x p c + y g c).

IV. SUMMARY

Due to a moratorium on disposal, a large buildup of assets with potential or actual FG usage has developed. This study develops a mathematical model based on economic criteria to compute the level of assets to be held in contingency retention to fill requisitions of FGs. The model formulation uses the tradeoff between the cost to hold assets in contingency retention and the proceeds to be gained by subsequently selling the assets to FGs. The model applies to assets held in contingency retention with a sufficient history of FG demand or anticipated failure rates similar to Navy failure rates to allow the development of a FG demand forecast. The model does not apply to assets applicable to weapons systems for which a FG demand forecast cannot be developed or estimated from Navy usage rates.

Several model constraints were developed to allow management greater flexibility in usage of the model and to allow for situations extraneous to the model such as the impact of limited shelf life of assets. The model was then evaluated by sensitivity analysis. This analysis showed that the model solution value becomes more sensitive as the price charged FGs for assets used to fill requisitions approaches the price received by the Navy on a 'fire sale', i.e., a sale of assets to FGs on other than a requisition basis, or disposal action. The model solution value is also most

sensitive to variations in all model parameters except x, (the fraction of assets disposed), as these parameters become small in magnitude. The analysis also showed that arbitrary decreases in the number of years of demand held in retention below the optimal level would have little impact for small decreases.

The model is solvable by a well known technique which may be incorporated into the appropriate UICP stratification program. Several model parameters have been introduced which can be estimated from empirical data or set by management policy.

In summary, the FMS economic retention model is a tool for computing the level of long supply assets which maximizes the economic return to the Navy. The model may be implemented in the UICP stratification process. Thus, an economic tool is now available to enable management to dispose of excess long supply assets.

Economic benefits accrue from reduced holding costs and increased returns from disposal. This is attained through more precise handling of Navy excess material with FMS application. To illustrate, the model was applied to a segment of Navy managed inventory with potential FMS demand. The value of the excess material held in contingency

retention by ASO is approximately $92M. Based on historical FG annual demand, it is estimated that $2.8M of the material will be requisitioned by FGs during the next year. Using the UICP policy parameter for holding cost rate of 21% and an estimated 3% disposal return rate for such material, the economic benefits are computed. It is estimated that selective reduction of the $92M excess in material with FMS application will reduce annual holding costs by $15M and return an estimated $2M to the U. S. Treasury through disposal action.

APPENDIX A: GLOSSARY

The following definitions were extracted from NAVSUP Publication 526, Foreign Military Sales Customer Supply System Guide, Washington, D. C., Navy Department, Naval Supply Systems Command:

1. COOPLOG (Cooperative Logistics). A support arrangement under which logistic support is provided to a foreign government through its participation in the U. S. Department of Defense Logistic System with reimbursement to the U. S. for support performed.

2. DRP (Direct Requisitioning Procedure). An open-end requisitioning case covering undefinitized spare parts for a specific weapons system. The FMS case is of specific duration, normally 12 months.

3. Accessorial Charges. A separate charge for packing, crating, port handling and loading, and transportation associated with the preparation and delivery of material.

4. Administrative Charges. Charges for expenses associated with the administration of the defense logistic system.

5. Case, FMS. A contractual sales agreement between the U. S. and an eligible foreign country documented by DD Form 1513. One FMS case designator is assigned for the purpose of identification, accounting, and data processing

for each accepted offer (DD Form 1513).

Definition 6 was obtained from Essentials of Managerial Finance, Second Edition, J. Fred Weston and Eugene F. Brigham, Holt, Rinehart and Winston, Inc., 1971.

6. Opportunity Cost. The expected return on the next best alternative foregone by choosing the best alternative.

Definition 7 was obtained from a FMSO Stratification Training Manual.

7. Contingency Retention. A requirement for a quantity of an item that should be retained even though it may not be economical to do so. The material is held, vice disposed, based on outside considerations to provide material for defense purposes.

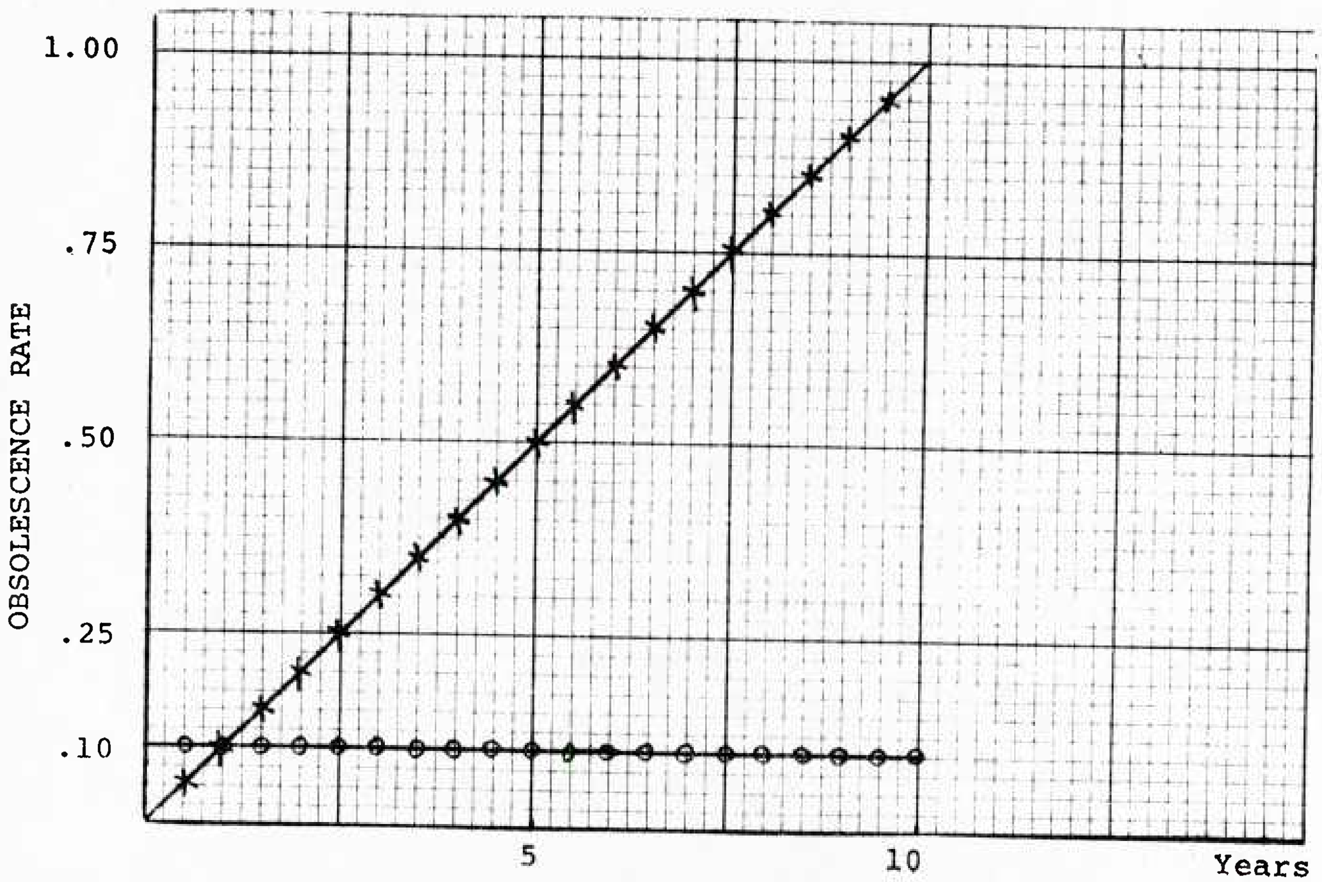
APPENDIX B: REFERENCES

1. NAVSUP ltr 04A1/RDS of 26 Apr 1977
2. DODINST 2140.1 of 17 Jun 1975
3. ALRAND Report 45 - "Inventory Control Manual - The Uniform Automated Data Processing System", U. S. Naval Supply Depot, Mechanicsburg, PA, 12 Apr 1965
4. Jennings, Walter, First Course in Numerical Methods, New York: The MacMillan Company, 1964

APPENDIX C: GRAPHICAL ANALYSIS

FIGURE I

Hypothetical illustrative example showing the uniform distribution of annual obsolescence and the cumulative distribution of annual obsolescence



LEGEND: ooooo represents the uniform distribution of obsolescence

xxxxx represents the cumulative distribution of obsolescence or the probability that an item will be obsolete after t years

FIGURE II

Hypothetical illustrative example showing the economic tradeoffs between storage, opportunity disposal costs and sale proceeds

LEGEND: Curve x-x-x represents dollar proceeds from sales of material to Foreign Governments

Curve O-O-O represents the storage costs plus the opportunity cost of not liquidating assets through disposal

Curve ⊗-⊗-⊗ represents the difference of sales proceeds minus holding costs and disposal opportunity cost or the expected net proceeds from sales of excess material to Foreign Governments

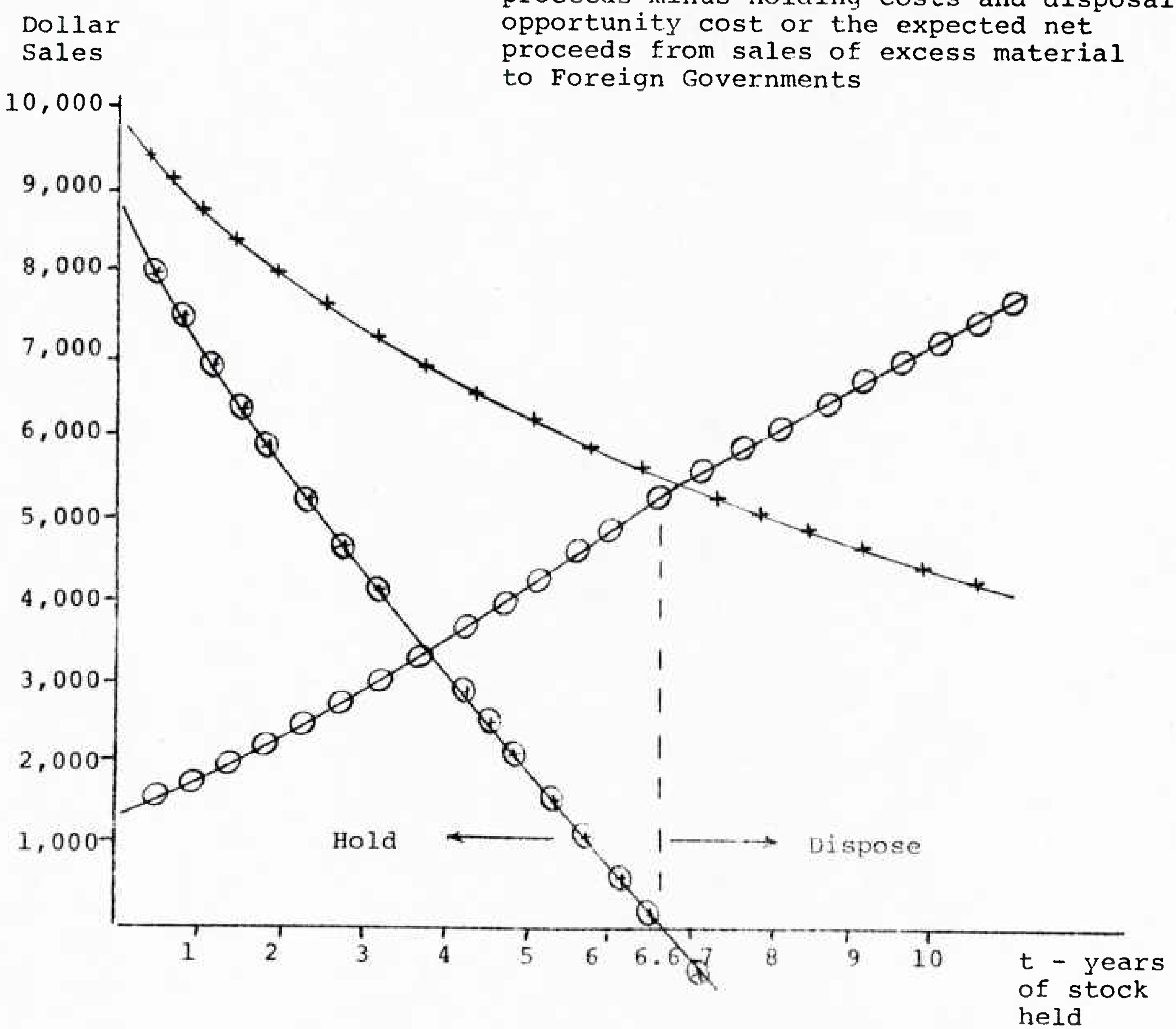


FIGURE III

Graphical solution of equation three for: i = .1, s = .01, a = .10, g = .5, p = .10, x = .90, y = .10, b = 1.00

LEGEND: Curve X-X-X represents the gross proceeds on a sale expressed as a fraction of standard price

Curve ⊙-⊙-⊙ represents the costs to hold material in inventory expressed as a fraction of standard price

Curve ●-●-● represents net proceeds expressed as a fraction of the standard price

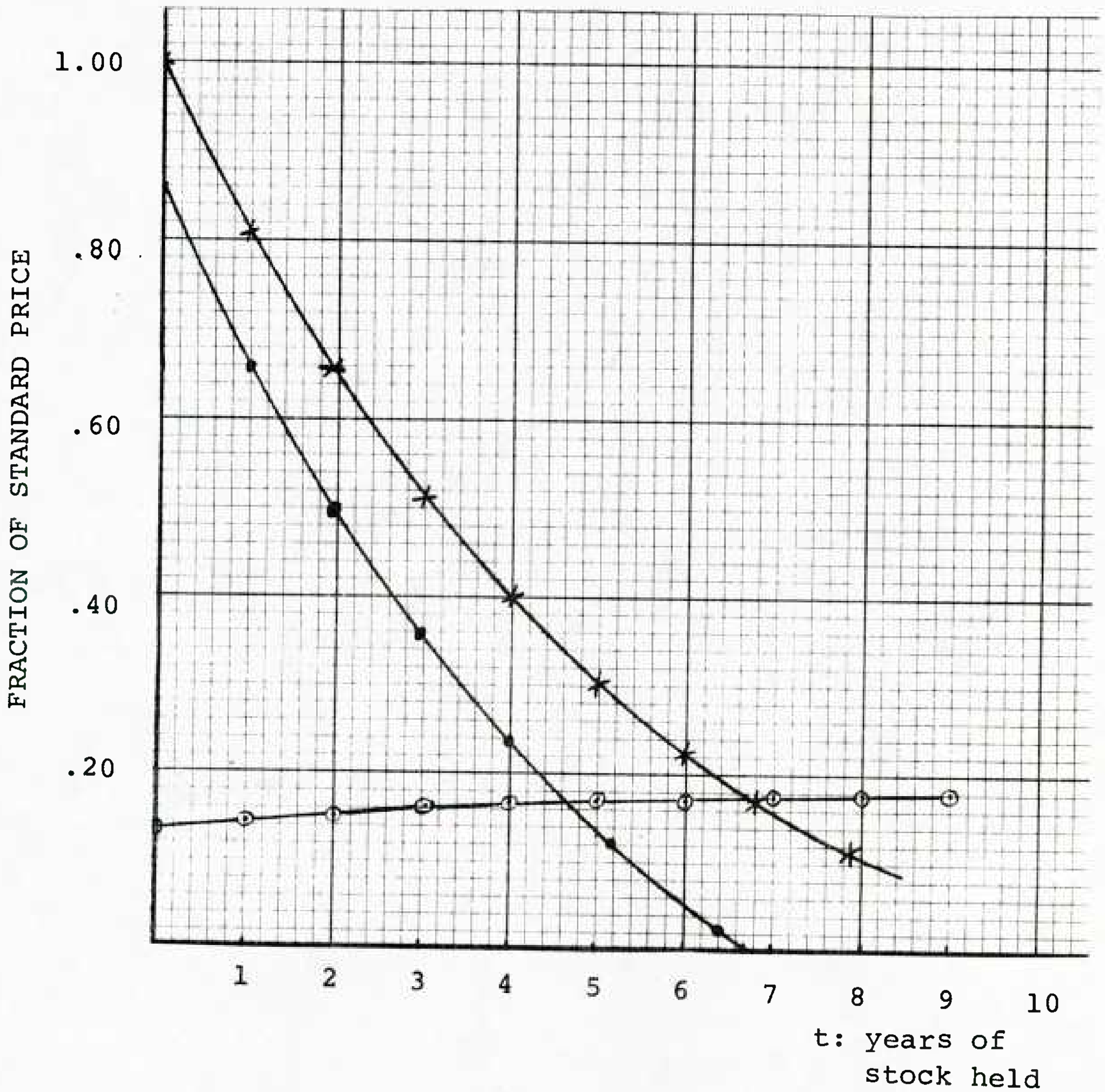


FIGURE IV

Graphical Representation of TABLE I of APPENDIX E

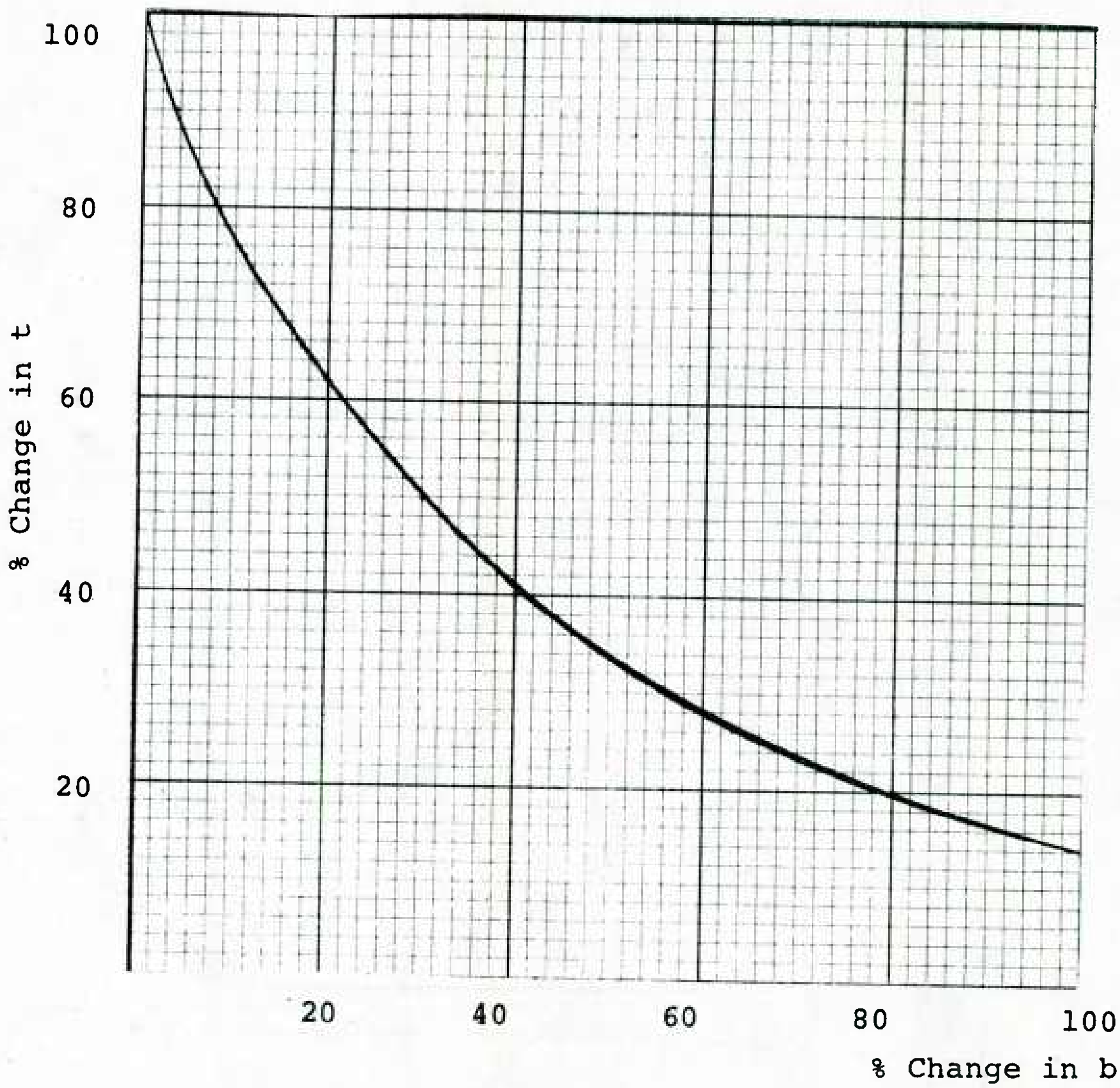


FIGURE V

Graphical representation of TABLES II, VIII, and IX of APPENDIX E

LEGEND: TABLE II curve X-X-X for b = 1.00 and p = .10
TABLE VIII curve O-O-O for b = .50 and p = .10
TABLE IX curve ⊗-⊗-⊗ for b = 1.00 and p = .40

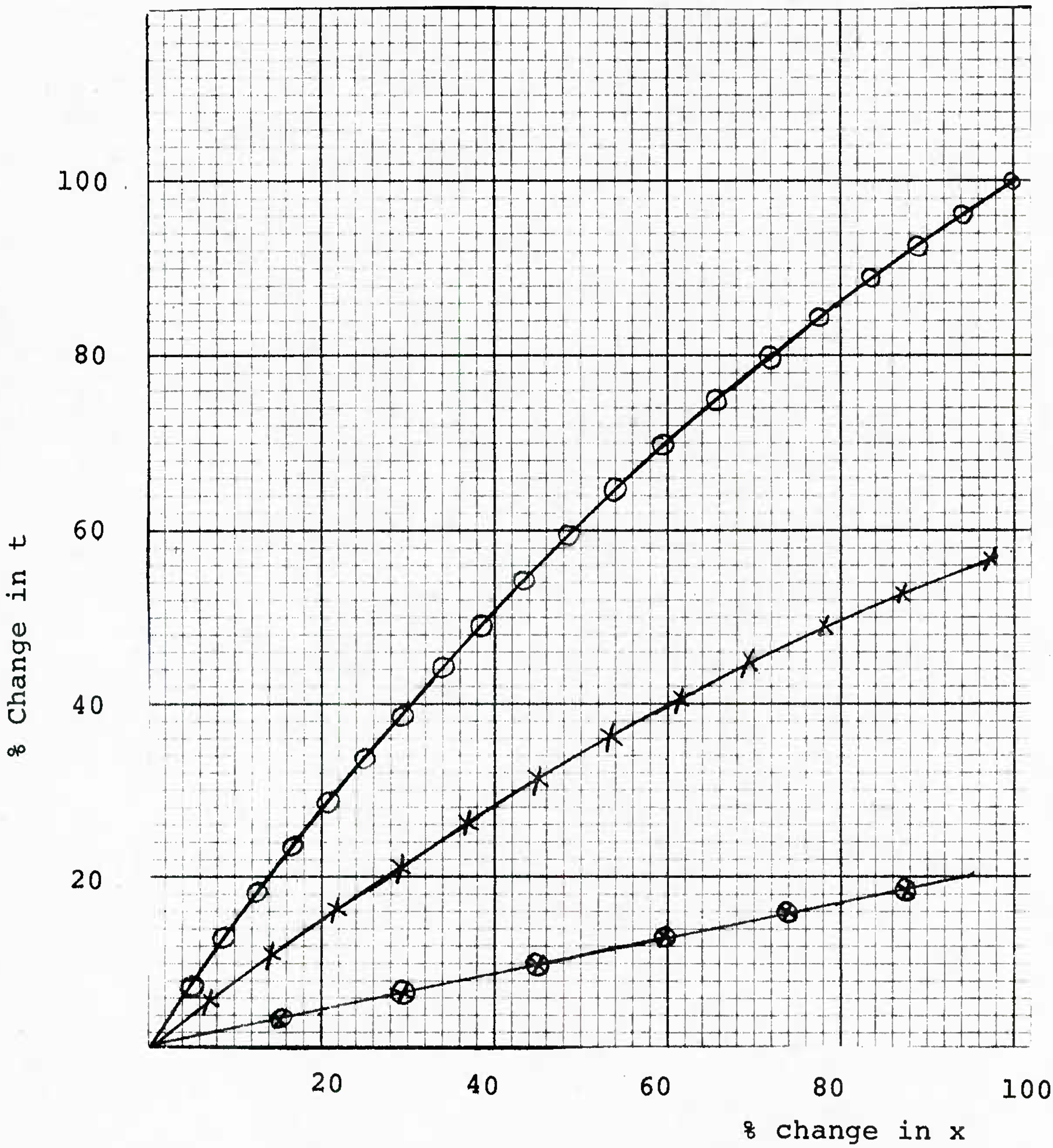


FIGURE VI

Graphical representation of TABLE III of APPENDIX E

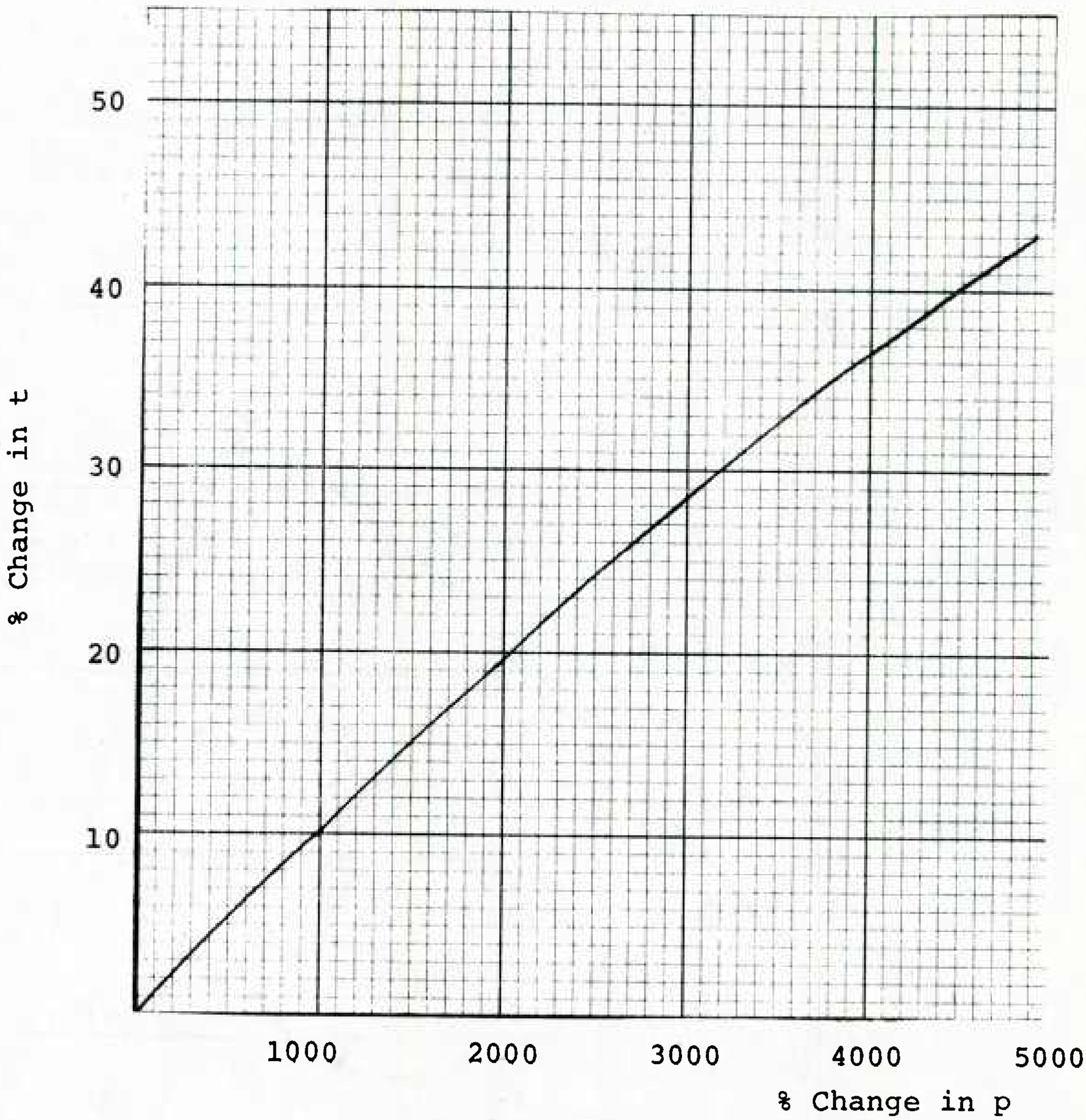


FIGURE VII

Graphical representation of TABLE IV of APPENDIX E

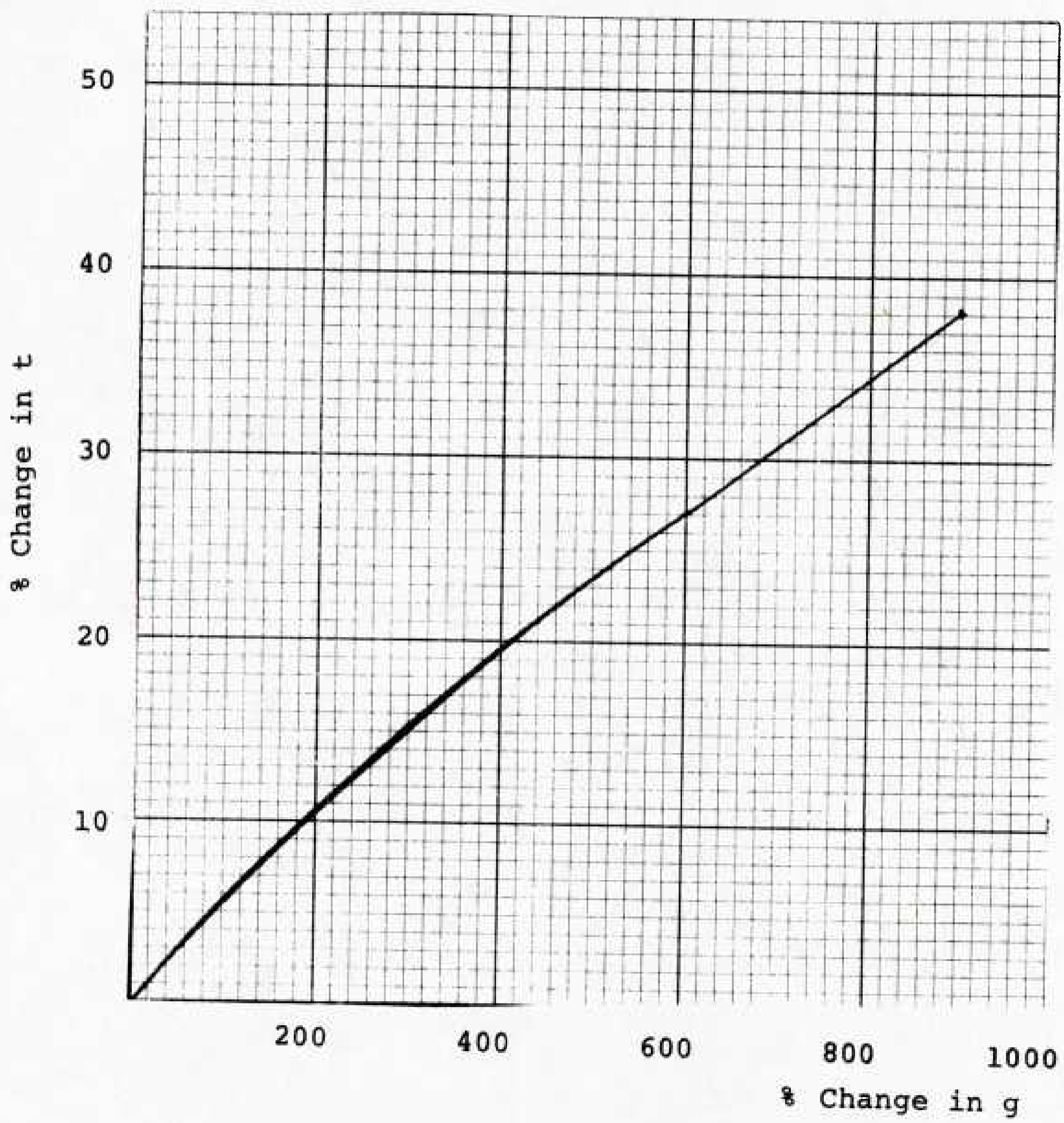


FIGURE VIII

Graphical representation of TABLE V of APPENDIX E

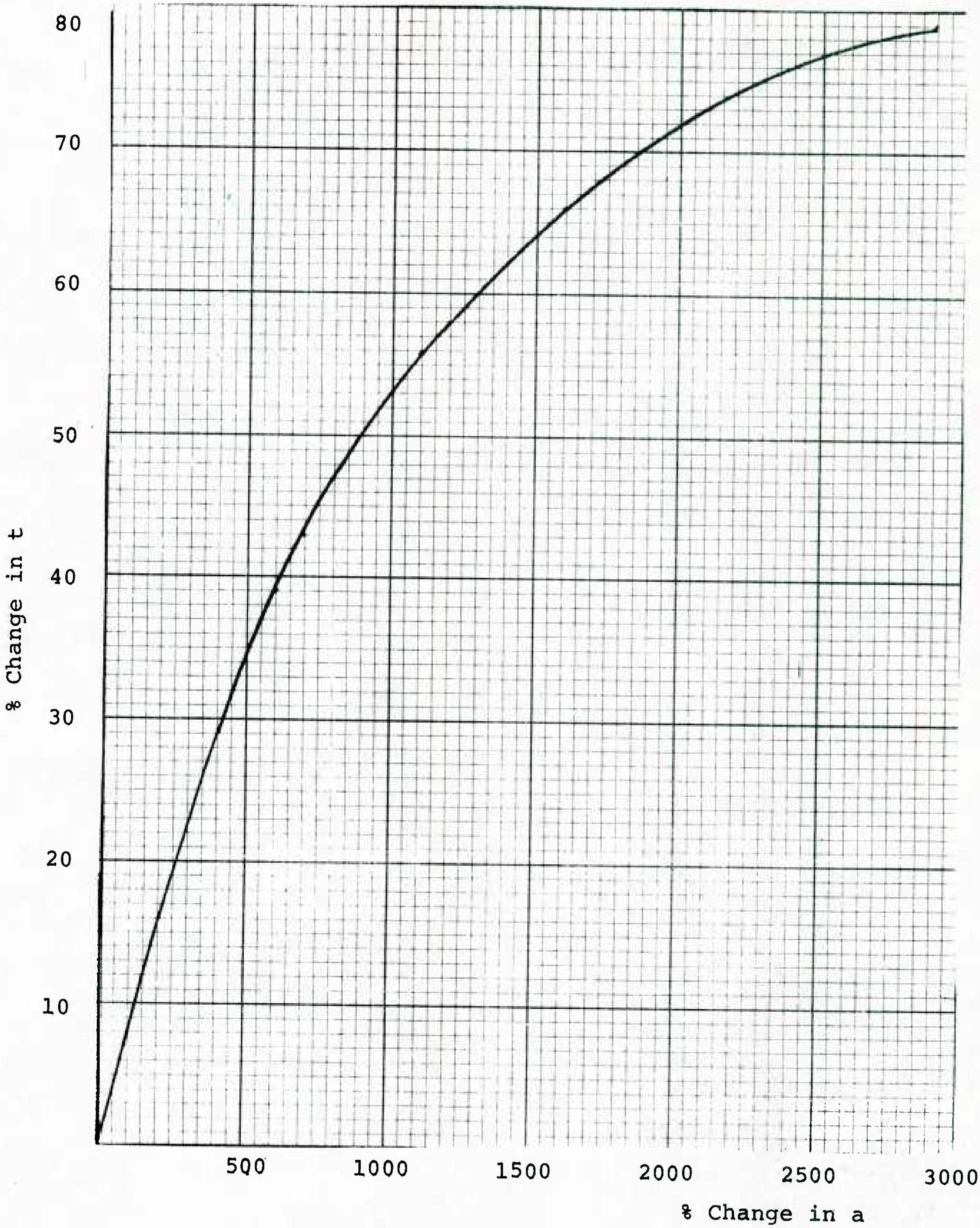


FIGURE IX

Graphical representation of TABLE VI of APPENDIX E

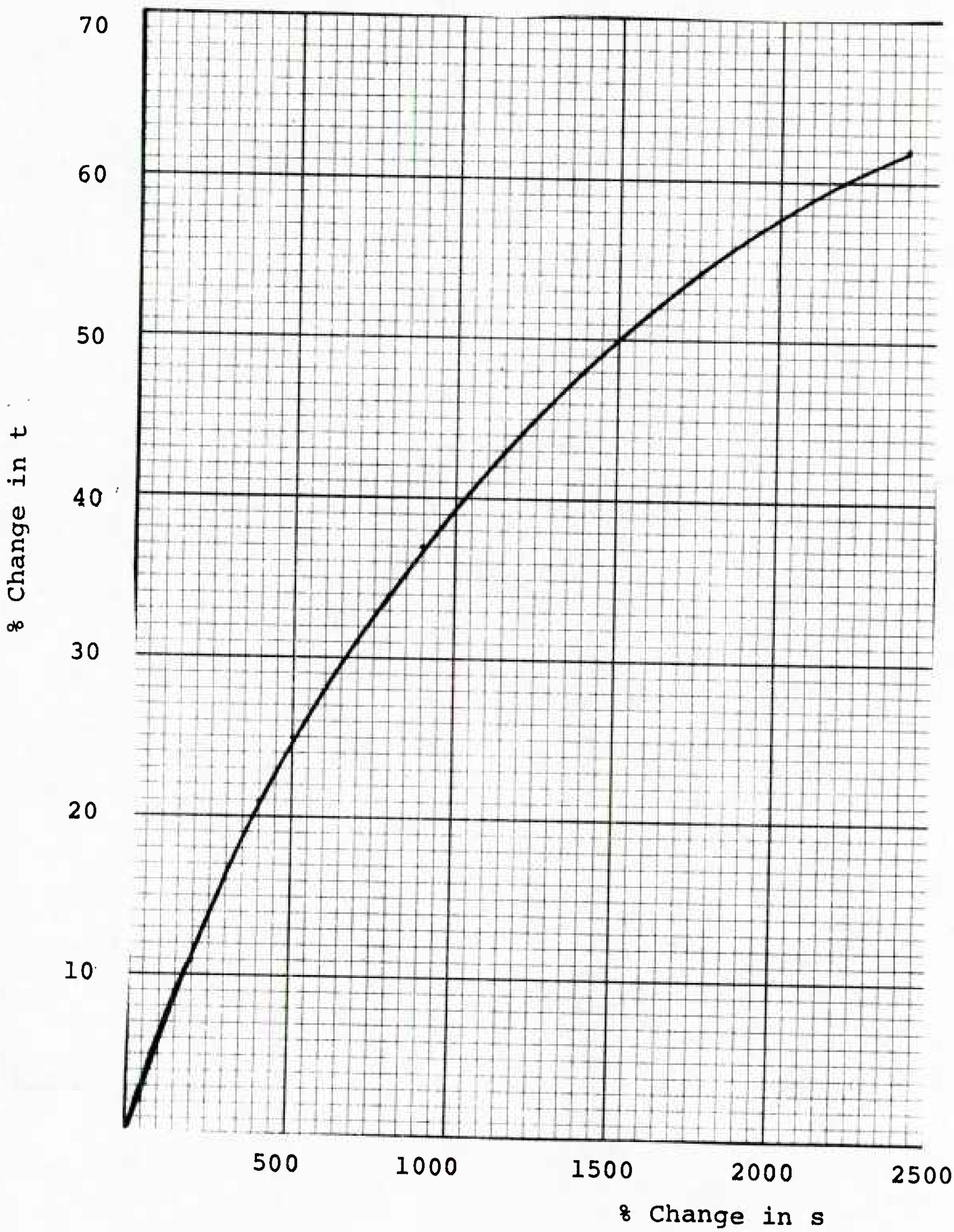
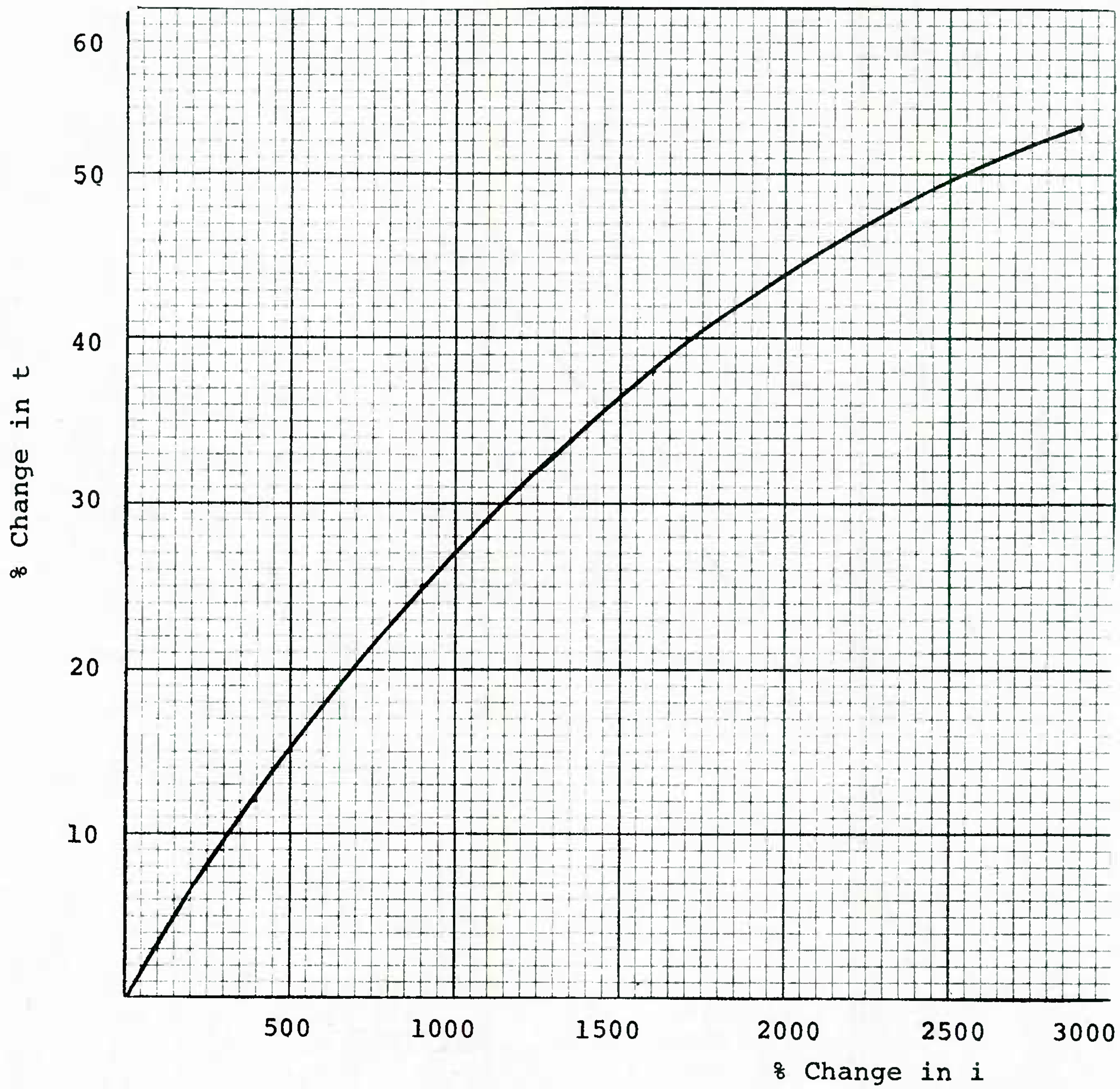


FIGURE X

Graphical representation of TABLE VII of APPENDIX E



APPENDIX D: MODEL SOLUTION TECHNIQUE

Reference 4 of APPENDIX B describes an iterative solution technique known as the method of linear interpolation (secant method). With this technique the relationship

(1)

$$t_{n+1} = \frac{1}{f(t_n)-f(t_{n-1})}\begin{vmatrix} t_{n-1} & f(t_{n-1}) \\ t_n & f(t_n) \end{vmatrix} = \frac{(t_{n-1})(f(t_n))-(t_n)(f(t_{n-1}))}{f(t_n)-f(f_{n-1})}$$

is used to find successive t_1 (or t_2) trial values. Consider the following example where:

$i = .10$

$s = .01$

$a = .10$

$g = .50$

$p = .10$

$x = .90$

$y = .10$

$b = 1.00$

The solution of t, i.e., t_1 or t_2, will be found when

(2)

$$f(t_n) = x\,p + y\,g + s\left[\frac{\left(\frac{1}{1+i}\right)^{t}\left[(1-ta)\ln\left(\frac{1}{1+i}\right) + a\right] - \left[\ln\left(\frac{1}{1+i}\right) + a\right]}{\left[\ln\left(\frac{1}{1+i}\right)\right]^{2}}\right]$$

$$- b\ (1-ta) \left(\frac{1}{1+i}\right)^{t} = 0$$

The solution process is as follows:

The process must be initiated by choosing t_{-1} and t_0 values and computing corresponding $f(t_{-1})$ and $f(t_0)$ values. With $t_{-1} = 0$:

$$f(t_{-1}) = (.9)(.1) + (.1)(.5) + (.01)$$

$$\left[\frac{\left(\frac{1}{1.1}\right)^{0}\left[(1-0)\ln\left(\frac{1}{1.1}\right) + .1\right] - \left[\ln\left(\frac{1}{1.1}\right) + .1\right]}{\left[\ln\left(\frac{1}{1.1}\right)\right]^{2}}\right] - \left(\frac{1}{1.1}\right)^{0}$$

$$f(t_{-1}) = -.86$$

Similarly, with $t_0 = 1$, $f(t_0) = -.760022002$

Using equation (1) above

$$t_{n+1} = t_1 = \frac{0(-.760022) - 1(-.86)}{-.760022 + .86} = 8.601892588$$

and $f(t_1) = -.115185971$. Continuing this process until $f(t_n)$ converges to zero to 6 decimal place accuracy yields the following results:

n	t_n	$f(t_n)$
-1	0.0	-.860000000
0	1.0	-.760022002
1	8.601892588	.115185971
2	7.601408811	.059671403
3	6.526010524	-.013767549
4	6.7276	.000685015
5	6.738155467	.002558153
6	6.723	.002062353

n	t_n	$f(t_n)$
7	6.659959661	-.00242757
8	6.694043751	.00023105
9	6.691081839	.000216225

The above solution of 6.69 years was found initially after 11 iterations. The solution value can be found with fewer iterations in subsequent applications of the solution technique if the model parameter values do not change drastically. The initial solution value can be stored and used as the t_0 value in successive applications. The t_{-1} value can be set to (t_0-1). With the above example, the t_{-1} value would be 5.69 and the t_0 value would be 6.69.

The UICP stratification program uses FORTRAN IV subroutines to perform mathematical computations. The above solution technique may also be incorporated into the appropriate stratification program via a FORTRAN IV subroutine.

Other iterative solution techniques are available such as the Newton-Raphson method. These techniques are similar to the method of linear interpolation explained above. All of these techniques are nearly equivalent in solving the model expression. The method of linear interpolation, however, is the simplest method to apply and is, therefore, recommended as the solution technique.

APPENDIX E: RESULTS OF SENSITIVITY ANALYSIS

FIGURE III of APPENDIX C illustrates the impact of variations in t on the model terms which represent costs and proceeds. The total economic advantage, i.e., the net proceeds, of holding a unit of an item for subsequent sale is found by subtracting the left side of the model from the right side. The left side represents costs incurred to hold assets and the right side represents gross proceeds on a sale. As FIGURE III of APPENDIX C shows, an asset sold immediately to a FG would earn 86% of the standard price. Increasing storage, interest, and obsolescence costs reduce these earnings per unit sold to virtually zero after 6.69 years.

FIGURE III of APPENDIX C shows that both gross and net proceeds of a sale become increasingly sensitive as t decreases. The steepness of the slopes of the curves in FIGURE III of APPENDIX C represent the sensitivity of the proceeds and costs to changes in t. The holding cost curve is virtually linear and relative flat illustrating minimal sensitivity of holding costs to variations in t.

In general, small reductions in the optimal number of years of demand held will have little impact. However, larger reductions will have increasing percentage effect on the

economic tradeoff.

TABLE I of this appendix illustrates the sensitivity of t as b is varied. The critical tradeoff here is the percentage b of the standard price to be gained by selling material held to fill FG requisitions versus the percentage of the standard price to be gained through disposal and/or a 'fire sale'. The latter proceeds would be x p + y g of the standard price or (.9)(.1) + (.1)(.5) = .14. As b approaches 14% of standard price, less stock should be held because it is not economical to incur holding costs to sell assets at a later date for a price that could be obtained now. As TABLE I below shows, t is 0 when b equals the critical value of 14%.

In general, t becomes increasingly sensitive to variations in b as b approaches the critical .14 value. These results can be seen graphically in FIGURE IV of APPENDIX C. The slope of the curve represents the sensitivity of t to changes in x.

TABLE II shows the results on t of varying x (and also y because x + y = 1). Each percentage change in x (with x increasing) results in a less than corresponding change in t. In general, t becomes less sensitive as x decreases. A graphical representation is shown in FIGURE V of APPENDIX C.

TABLE VIII below shows the impact on t by varying x

with b reduced to .5. Generally, the same basic pattern emerges as in the case depicted by TABLE II of APPENDIX E. However, in this case, t has become smaller and less sensitive overall to changes in x. FIGURE V of APPENDIX C graphically shows the TABLE VIII case compared to the TABLE II case.

TABLE IX below shows the impact on t by varying x with p = .4 and z = 1.0. Again, the same general pattern emerges as was the case with TABLES II and VIII. However, t is smaller in this case and t is not as sensitive to change. FIGURE V of APPENDIX C is a graphical representation of TABLE IX values.

TABLE III shows that t becomes less sensitive to changes in p as p gets larger and approaches g. The p values were held less than or equal to the g values because if p were greater than g, every asset should be disposed rather than sold on a 'fire sale'. This condition would be inconsistent with the .5 value of x and y. FIGURE VI of APPENDIX C graphically depicts the TABLE III case.

TABLE IV below shows basically the same results as TABLE III. Therefore, varying either p or g will have similar results on t. FIGURE VII of APPENDIX C is a graphical representation of the results of TABLE IV.

TABLE V below shows that t is more sensitive to changes in a when a is small. As a increases, t becomes progressively less sensitive. FIGURE VIII of APPENDIX C graphically depicts this case.

TABLE VI below and FIGURE IX of APPENDIX C show that t is most sensitive to changes in s when s is small. TABLE VII below and FIGURE X of APPENDIX C show that t is most sensitive to changes in i when i is small.

RESULTS OF SENSITIVITY ANALYSIS

TABLE I

Sensitivity of t with b varied

	i	s	a	g	p	x	y	b	%Δb	t	%Δt
Benchmark	.1	.01	.1	.5	.1	.9	.1	1.00	0	6.69	0
	.1	.01	.1	.5	.1	.9	.1	.99	1	6.67	0
	.1	.01	.1	.5	.1	.9	.1	.97	3	6.62	1
	.1	.01	.1	.5	.1	.9	.1	.90	10	6.43	4
	.1	.01	.1	.5	.1	.9	.1	.75	25	5.93	11
	.1	.01	.1	.5	.1	.9	.1	.50	50	4.69	30
	.1	.01	.1	.5	.1	.9	.1	.40	60	3.93	41
	.1	.01	.1	.5	.1	.9	.1	.25	75	2.21	67
	.1	.01	.1	.5	.1	.9	.1	.24	76	2.05	69
	.1	.01	.1	.5	.1	.9	.1	.20	80	1.36	80
	.1	.01	.1	.5	.1	.9	.1	.15	85	.26	96
	.1	.01	.1	.5	.1	.9	.1	.14	86	0	100

RESULTS OF SENSITIVITY ANALYSIS

TABLE II

Sensitivity of t with x and y varied and g equal to 1.00

	i	s	a	g	p	x	%Δx	y	%Δy	b	t	%Δt
Benchmark	.1	.01	.1	.5	.1	.99	0	.01	0	1.00	7.22	0
	.1	.01	.1	.5	.1	.98	1	.02	100	1.00	7.16	1
	.1	.01	.1	.5	.1	.97	2	.03	200	1.00	7.10	2
	.1	.01	.1	.5	.1	.90	9	.10	900	1.00	6.69	7
	.1	.01	.1	.5	.1	.75	24	.25	2400	1.00	5.91	18
	.1	.01	.1	.5	.1	.71	28	.29	2800	1.00	5.71	21
	.1	.01	.1	.5	.1	.50	49	.50	4900	1.00	4.79	34
	.1	.01	.1	.5	.1	.25	75	.75	7400	1.00	3.85	47
	.1	.01	.1	.5	.1	.10	90	.90	8900	1.00	3.34	54
	.1	.01	.1	.5	.1	.01	99	.99	9800	1.00	3.06	58

RESULTS OF SENSITIVITY ANALYSIS

TABLE III

Sensitivity of t with p varied

	i	s	a	g	p	%Δp	x	y	b	t	%Δt
Benchmark	.1	.01	.1	.5	.01	0	.5	.5	1.00	5.27	0
	.1	.01	.1	.5	.02	100	.5	.5	1.00	5.21	1
	.1	.01	.1	.5	.10	900	.5	.5	1.00	4.79	9
	.1	.01	.1	.5	.25	2400	.5	.5	1.00	4.07	23
	.1	.01	.1	.5	.50	4900	.5	.5	1.00	3.03	43

TABLE IV

Sensitivity of t with g varied

	i	s	a	g	%Δg	p	x	y	b	t	%Δt
Benchmark	.1	.01	.1	.05	0	.1	.5	.5	1.00	7.69	0
	.1	.01	.1	.10	100	.1	.5	.5	1.00	7.28	5
	.1	.01	.1	.20	300	.1	.5	.5	1.00	6.59	15
	.1	.01	.1	.35	600	.1	.5	.5	1.00	5.61	27
	.1	.01	.1	.50	900	.1	.5	.5	1.00	4.79	38

RESULTS OF SENSITIVITY ANALYSIS

TABLE V

Sensitivity of t with a varied

	i	s	a	%Δa	g	p	x	y	b	t	%Δt
Benchmark	.1	.01	.01	0	.5	.1	.5	.5	1.00	9.64	0
	.1	.01	.02	100	.5	.1	.5	.5	1.00	8.83	8
	.1	.01	.03	200	.5	.1	.5	.5	1.00	8.09	16
	.1	.01	.04	300	.5	.1	.5	.5	1.00	7.43	23
	.1	.01	.05	400	.5	.1	.5	.5	1.00	6.84	29
	.1	.01	.06	500	.5	.1	.5	.5	1.00	6.33	34
	.1	.01	.07	600	.5	.1	.5	.5	1.00	5.87	39
	.1	.01	.08	700	.5	.1	.5	.5	1.00	5.47	43
	.1	.01	.09	800	.5	.1	.5	.5	1.00	5.11	47
	.1	.01	.10	900	.5	.1	.5	.5	1.00	4.79	50
	.1	.01	.11	1000	.1	.1	.5	.5	1.00	4.51	53
	.1	.01	.12	1100	.5	.1	.5	.5	1.00	4.25	56
	.1	.01	.13	1200	.5	.1	.5	.5	1.00	4.02	58
	.1	.01	.14	1300	.5	.1	.5	.5	1.00	3.81	60
	.1	.01	.15	1400	.5	.1	.5	.5	1.00	3.63	62
	.1	.01	.17	1600	.5	.1	.5	.5	1.00	3.30	66
	.1	.01	.20	1900	.5	.1	.5	.5	1.00	2.79	71
	.1	.01	.25	2400	.5	.1	.5	.5	1.00	2.41	75
	.1	.01	.30	2900	.5	.1	.5	.5	1.00	2.06	79

RESULTS OF SENSITIVITY ANALYSIS

TABLE VI

Sensitivity of t with s varied

	i	s	%Δs	a	g	p	x	y	b	t	%Δt
Benchmark	.1	.01	0	.1	.5	.1	.5	.5	1.0	4.79	0
	.1	.02	100	.1	.5	.1	.5	.5	1.0	4.50	6
	.1	.03	200	.1	.5	.1	.5	.5	1.0	4.25	11
	.1	.04	300	.1	.5	.1	.5	.5	1.0	4.01	16
	.1	.05	400	.1	.5	.1	.5	.5	1.0	3.81	21
	.1	.06	500	.1	.5	.1	.5	.5	1.0	3.62	25
	.1	.07	600	.1	.5	.1	.5	.5	1.0	3.44	28
	.1	.08	700	.1	.5	.1	.5	.5	1.0	3.29	31
	.1	.09	800	.1	.5	.1	.5	.5	1.0	3.15	34
	.1	.10	900	.1	.5	.1	.5	.5	1.0	3.01	37
	.1	.15	1400	.1	.5	.1	.5	.5	1.0	2.49	48
	.1	.25	2400	.1	.5	.1	.5	.5	1.0	1.84	62

RESULTS OF SENSITIVITY ANALYSIS

TABLE VII

Sensitivity of t with i varied

	i	%Δi	s	a	g	p	x	y	b	t	%Δt
Benchmark	.01	0	.01	.1	.5	.1	.5	.5	1.00	6.35	0
	.02	100	.01	.1	.5	.1	.5	.5	1.00	6.15	3
	.03	200	.01	.1	.5	.1	.5	.5	1.00	5.96	6
	.04	300	.01	.1	.5	.1	.5	.5	1.00	5.77	9
	.05	400	.01	.1	.5	.1	.5	.5	1.00	5.59	12
	.06	500	.01	.1	.5	.1	.5	.5	1.00	5.41	15
	.07	600	.01	.1	.5	.1	.5	.5	1.00	5.25	17
	.08	700	.01	.1	.5	.1	.5	.5	1.00	5.09	20
	.09	800	.01	.1	.5	.1	.5	.5	1.00	4.94	22
	.10	900	.01	.1	.5	.1	.5	.5	1.00	4.79	25
	.11	1000	.01	.1	.5	.1	.5	.5	1.00	4.65	27
	.12	1100	.01	.1	.5	.1	.5	.5	1.00	4.52	29
	.13	1200	.01	.1	.5	.1	.5	.5	1.00	4.40	31
	.14	1300	.01	.1	.5	.1	.5	.5	1.00	4.28	33
	.15	1400	.01	.1	.5	.1	.5	.5	1.00	4.17	34
	.16	1500	.01	.1	.5	.1	.5	.5	1.00	4.06	36
	.17	1600	.01	.1	.5	.1	.5	.5	1.00	3.96	38
	.18	1700	.01	.1	.5	.1	.5	.5	1.00	3.86	39
	.19	1800	.01	.1	.5	.1	.5	.5	1.00	3.77	41
	.20	1900	.01	.1	.5	.1	.5	.5	1.00	3.69	42
	.25	2400	.01	.1	.5	.1	.5	.5	1.00	3.31	48
	.30	2900	.01	.1	.5	.1	.5	.5	1.00	3.00	53

RESULTS OF SENSITIVITY ANALYSIS

TABLE VIII

Sensitivity of t with x and y varied and b equal to .5 and p equal to .1

	i	s	a	g	p	x	%Δx	y	%Δy	b	t	%Δt
Benchmark	.1	.01	.1	.5	.1	.99	0	.01	0	.5	5.44	0
	.1	.01	.1	.5	.1	.98	1	.02	100	.5	5.35	2
	.1	.01	.1	.5	.1	.97	2	.03	200	.5	5.26	3
	.1	.01	.1	.5	.1	.90	9	.10	900	.5	4.69	14
	.1	.01	.1	.5	.1	.75	24	.25	2400	.5	3.63	33
	.1	.01	.1	.5	.1	.50	49	.50	4900	.5	2.18	60
	.1	.01	.1	.5	.1	.25	75	.75	7400	.5	1.00	82
	.1	.01	.1	.5	.1	.10	90	.90	8900	.5	.38	93
	.1	.01	.1	.5	.1	.01	99	.99	9800	.5	.04	99

RESULTS OF SENSITIVITY ANALYSIS

TABLE IX

Sensitivity of t with x and y varied and b equal to 1.00 and p equal to .4

	i	s	a	g	p	x	%Δx	y	%Δy	b	t	%Δt
Benchmark	.1	.01	.1	.5	.4	.99	0	.01	0	1.0	3.84	0
	.1	.01	.1	.5	.4	.98	1	.02	100	1.0	3.83	0
	.1	.01	.1	.5	.4	.97	2	.03	200	1.0	3.82	0
	.1	.01	.1	.5	.4	.90	9	.10	900	1.0	3.76	2
	.1	.01	.1	.5	.4	.75	24	.25	2400	1.0	3.63	5
	.1	.01	.1	.5	.4	.50	49	.50	4900	1.0	3.42	11
	.1	.01	.1	.5	.4	.25	75	.75	7400	1.0	3.22	16
	.1	.01	.1	.5	.4	.10	90	.90	8900	1.0	3.10	19
	.1	.01	.1	.5	.4	.01	99	.99	9800	1.0	3.03	21